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Generator modules of segmented thermoelements

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ABSTRACT

Physical factors affecting the efficiency of segmented generator thermoelements are analyzed. The optimal control theory are used for development of a sufficiently precise method for a computerize design of generator modules of segmented thermoelements. The optimal thermoelectric parameters of BiTe-based materials for single- and double-segmented thermoelements are determined. The experimental generator samples have been made of such materials and their characteristics measured. The efficiency of modules of double-segmented legs has been found to be about 7.5% and exceed the power efficiency of homogeneous material generators.

1. Introduction

The efficiency of thermoelectric power generators varies depending on the figure of merit of materials and the operating temperature difference of thermoelements. Present-day thermoelectric materials have maximum value of figure of merit Z in a relatively narrow temperature interval. These intervals are considerably smaller than the operating range where thermoelectric generators can function. Therefore, single-stage generator modules of homogeneous materials can work adequately only under low-temperature differences (100–300 K), which restricts their efficiency. For example, maximum efficiency of commercial modules of BiTe-based materials produced by different companies worldwide does not exceed 5.8% [1].

One of the ways to improve the efficiency of generator modules is to use segmented legs with series-connected materials having different values of the Seebeck coefficient α, electric conductivity σ, and thermal conductivity κ. Segmented thermoelement for generator was patented in 1962 [2]. The main concept of the patent is to improve the efficiency by using materials with maximum figure of merit corresponding to the operating temperature range of individual sections.

The sections can be made of identical thermoelectric material, however, with different impurity concentration optimal for the operating temperature range of each individual section. Such segmented thermoelements are called concentration ones [3].

Thermoelements whose sections are made of different thermoelectric materials have much wider operating temperature ranges, hence the increased efficiency. The main requirements to manufacturing of such thermoelements include overcoming technological difficulties related to chemical and thermal compatibility of materials.

However, apart from these technological difficulties, the problem of electric compatibility of sections arises in the development of segmented thermoelements. For the first time this problem was discussed in [3]. Under condition of series connection of electric power sources (sections) the ratio of their electromotive force (EMF) to internal resistance should be constant: $E_k/r_k = \text{const}$, otherwise, there will be parasitic losses of electric power on internal resistances of sections with high resistance value and low EMF. Hence, current passing through the segments of thermoelement legs must be equal. At the same time, current value has to be close to its optimal value in each section. For maximum efficiency mode optimal current can be written as

$$I = I_k^{opt} = \frac{\alpha_k \Delta T_k}{r_k(1+\sqrt{1+Z_k\overline{\overline{T_k}}})} = \frac{Z_k Q_k^{\lambda}}{\alpha_k(1+\sqrt{1+Z_k\overline{\overline{T_k}}})} = \text{const.} \quad (1)$$

where $\overline{T}_k$ is the average temperature of kth segment. As noted in [3], in the approximation that conduction heat Q_k^{λ} through adjacent sections are little different, equality (1) holds only in the case when the ratio of figure of merit Z_k to the Seebeck coefficient α_k of segment materials is the same: $Z_k/\alpha_k \approx \text{const}$. It is noteworthy that for the mode of maximum efficiency of segmented generator a more precise condition of matching section materials is equality for each segment of the so-called compatibility factor s:

$$s = \frac{Z_k}{(1+\sqrt{1+Z_k\overline{T}_k})\alpha_k}. \quad (2)$$

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Nomenclature

C	impurity concentration (cm^{-3})
H	Hamiltonian function
I	electrical current (A)
i	density of electrical current (A cm^{-2})
j	specific density of electrical current (A cm^{-1})
l	height of the thermoelectric legs (cm)
N	quantity of segments
n	number of thermoelements in a module
Q	power of heat flux (W)
q	specific heat flux (W A^{-1})
r	contact resistance (Ω cm^2)
S	cross-section area of the thermoelectric legs (cm^2)
s	compatibility factor
T	temperature (K)
T_h	hot junction temperature (K)
T_c	cold junction temperature (K)
U	voltage (V)
W	power (W)
x	coordinate (cm)
y	dimensionless coordinate
Z	material figure of merit (K^{-1})

Greek symbols

α	Seebeck coefficient (V K^{-1})
κ	thermal conductivity (W cm^{-1} K^{-1})
σ	electrical conductivity (Ω^{-1} cm^{-1})
σ_0	electrical conductivity at 300 K (Ω^{-1} cm^{-1})
ψ	components of vector-function of pulses
η	conversion efficiency (%)

Subscripts

n	n-type thermoelectric material
p	p-type thermoelectric material
k	number of segment

It is evident that exactly this condition follows from (1) in the approximation $Q_k^{\lambda} \approx$ const. In case of considerable deviation of s values, the sections operate in the modes that are drastically different from the optimal, and the efficiency of such thermoelement is reduced. A similar condition for compatibility of segmented generator thermoelement materials was derived on the basis of somewhat different approach in [4].

A segmented thermoelement has a number of series-connected junctions. Its generated thermoEMF is defined as a sum of partial thermoEMFs of electric power sources (sections) connected in series with their own signs [5]. A sign and value of partial thermoEMFs are determined by a sign and value of difference in the Seebeck coefficients at the interface between the sections. In case of like signs of partial thermoEMFs the generated thermoEMF and accordingly conversion efficiency are increased, otherwise their values are reduced.

There is one more factor governing the efficiency of segmented thermoelements. It is the value of contact resistance at the boundary between the sections, as well as between the legs and connecting plates. In case of increased values of contact resistances the generated electric power is lowered by the magnitude of Joule heat released therein and the efficiency is reduced.

Thus, the efficiency of segmented thermoelements is affected by four main factors:

- the figure of merit of section materials, or, more precisely, the temperature dependences of their basic thermoelectric parameters α, σ, and κ;
- the compatibility factor of section materials s;
- the sign and value of difference in the Seebeck coefficients at the interface between the sections; and
- the values of contact resistances.

Therefore, optimization at the stage of designing a generator module of segmented thermoelements that lies in a substantiated selection of thermoelectric materials, operating temperature ranges, and optimal geometry of sections, must take into account matched influence of these factors.

This is a complicated challenge that would not be solved by analytical methods. Review of the literature has shown that for its solution only approximate methods were used [3,6,7–11]. The basic approximation include the following concepts:

- use instead of real temperature dependences of constant values of material properties α, σ, and κ averaged by temperature [3,6] or volume [7,8];
- use for design of approximate algebraic heat balance equations under given temperatures at the interface of sections [3,6];
- specific heat flow caused by thermal conductivity in all segments is taken as a constant value [3,9];
- temperatures between sections are not optimized, but approximately determined on the basis of temperature dependences of Z or ZT of thermoelectric materials [3,6] or from another considerations [9]; and
- influence of contact resistances is not taken into consideration [3,10,11].

Using approximate design methods can explain to a certain extent the fact that experimental studies of the first variants of elaborated segmented generator thermoelements demonstrated considerably worse characteristics that expected theoretically [10,12].

In this connection the relevant task is to elaborate and use a more realistic model for the design and optimization of generator modules based on segmented thermoelements that would allow for the effect of the above factors.

The purpose of this work is to develop a sufficiently precise method for a computerize design of generator modules of segmented thermoelements, as well as to calculate power characteristics of such modules and compare them to experimental values.

Ref. [13] describes a method for optimization of segmented thermoelements for coolers based on the use of optimal control theory. This theory was also utilized for optimization of generator modules of traditional single-segmented thermoelements [14] and for design of functionally graded materials and generator modules on their basis [15]. Optimal control methods allow a more precise optimization of thermoelectric modules obviating the need for the above approximations.

In this paper optimal control theory is generalized for precise computer design of generator modules with segmented legs. The method allows designing optimal segmented thermoelements with different materials of sections, as well as concentration thermoelements. When different materials are used, their characteristics should be matched according to condition (2). When designing concentration segmented thermoelements, the method makes it possible to determine for each section optimal impurity

concentrations matched with their optimal geometric dimensions and operating temperature ranges.

2. Design of generator modules of segmented thermoelements

The physical model of segmented thermoelement used for construction of a computerized design method is shown in Fig. 1. The legs of n- and p-type are composed of N_n and N_p segments, respectively. In the general case, sections can be made of different thermoelectric materials. Temperatures on the heat-releasing and heat-absorbing surfaces of thermoelement are fixed, the lateral surface is adiabatically insulated. The model takes into account the availability of contact resistances r_n and r_p near the heat-releasing and heat-absorbing junctions and at the section interfaces of n- and p-legs, respectively.

The system of equations describing thermal and electrical processes occurring in the infinitesimal part dx of each kth segment of n- and p-type legs is as follows [16]:

$$\left.\begin{aligned} \frac{dT}{dx} &= -\frac{\alpha_k i}{\kappa_k}T - \frac{i}{\kappa_k}q, \\ \frac{dq}{dx} &= \frac{\alpha_k^2 i}{\kappa_k}T + \frac{\alpha_k i}{\kappa_k}q + \frac{i}{\sigma_k}, \end{aligned}\right\}_{n,p} \begin{aligned} & k = 1,\ldots,N_{n,p} \\ & x_{k-1} \leqslant x \leqslant x_k \end{aligned}. \tag{3}$$

Specific heat flux q and electric current density i are determined by relationships

$$q = \frac{Q}{I}, \quad i_{n,p} = \frac{I}{S_{n,p}}, \tag{4}$$

where Q is the power of heat flux passing through thermoelement leg, I the value of generated current, $S_{n,p}$ are the cross-sectional areas of n- and p-type thermoelement legs. Necessary for solving (3), the multipoint boundary conditions for each leg are written as follows:

$$\begin{aligned} & T_{n,p}(0) = T_c, \quad T_{n,p}(l) = T_h, \\ & T_{n,p}(x_k^+) = T_{n,p}(x_k^-), \quad q_{n,p}(x_k^+) = q_{n,p}(x_k^-) + \frac{r_{n,p}}{S_{n,p}}I, \end{aligned} \tag{5}$$

where indexes "–" and "+" refer to the values of functions immediately to the left and to the right from the boundary x_k of neighboring sections, l is the height of thermoelement legs.

If a segmented thermoelement is designed of predetermined different materials of sections, then each segment material parameters α_k, σ_k, and κ_k are functions of temperature: $\alpha_k = \alpha_k(T)$, $\sigma_k = \sigma_k(T)$, and $\kappa_k = \kappa_k(T)$. In case of designing a concentration element, α_k, σ_k, and κ_k depend on temperature and concentration C_k of impurity in this material: $\alpha_k = \alpha_k(C_k,T)$, $\sigma_k = \sigma_k(C_k,T)$, and $\kappa_k = \kappa_k(C_k,T)$. A general variant is also possible here: sections of different materials with temperature- and concentration-dependent parameters α_k, σ_k, and κ_k.

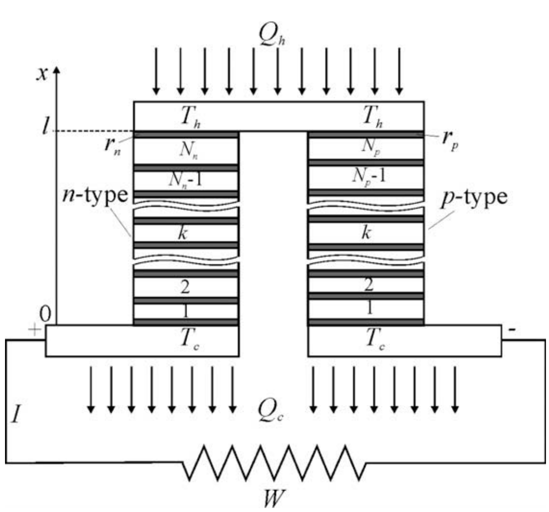

Fig. 1. Schematic of segmented generator thermoelement.

The objective in optimal designing a generator thermoelement is to determine the values of parameters: current density $i_{n,p}$ in the legs, height of sections $l_k = x_k - x_{k-1}$, impurity concentration C_k in materials of each section, such that thermoelement efficiency reaches the largest value. The efficiency is determined through specific heat flows on the cold and hot thermoelement junctions $q(0)$ and $q(l)$ as follows:

$$\eta = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{q(0)}{q(l)}, \tag{6}$$

where

$$\begin{aligned} & q(0) = q_n(0) + q_p(0) - I\left(\frac{r_n}{S_n} + \frac{r_p}{S_p}\right), \\ & q(l) = q_n(l) + q_p(l) + I\left(\frac{r_n}{S_n} + \frac{r_p}{S_p}\right). \end{aligned} \tag{7}$$

This problem is solved by creation of special computer program based on the use of mathematical optimal control theory. The essence of solution method lies in assigning certain initial approximations of the target values for parameters. The method reduces to a search for optimal values of these parameters by calculation of efficiency for their various values different from the assigned initial ones.

To use the method, the system of Eq. (3) is written in dimensionless coordinates $y_k = \frac{x - x_{k-1}}{l_k}$ and $x_{k-1} \leqslant x \leqslant x_k$ as follows:

$$\left.\begin{aligned} \frac{dT}{dy_k} &= -\frac{\alpha_k j_k}{\kappa_k}T - \frac{j_k}{\kappa_k}q, \\ \frac{dq}{dy_k} &= \frac{\alpha_k^2 j_k}{\kappa_k}T + \frac{\alpha_k j_k}{\kappa_k}q + \frac{j_k}{\sigma_k}, \end{aligned}\right\}_{n,p} \begin{aligned} & k = 1,\ldots,N_{n,p} \\ & 0 \leqslant y_k \leqslant 1 \end{aligned}, \tag{8}$$

where $j_k = i \cdot l_k$, l_k is the height of kth segment. Taking into account that

$$l = \sum_{k=1}^{N} l_k = \sum_{k=1}^{N} \frac{j_k S}{I},$$

the boundary conditions (5) for n- and p-legs will be re-written as

$$\begin{aligned} & T_{n,p}^{(1)}(0) = T_c, \quad T_{n,p}^{(N_{n,p})}(1) = T_h, \\ & T_{n,p}^{(k+1)}(0) = T_{n,p}^{(k)}(1), \quad q_{n,p}^{(k+1)}(0) = q_{n,p}^{(k)}(1) + \frac{r_{n,p}}{l}\sum_{k=1}^{N_{n,p}} j_k, \\ & \quad k = 1,\ldots,N_{n,p} - 1. \end{aligned} \tag{9}$$

Instead of maximum thermoelement efficiency it is convenient to seek for minimum functional J in a logarithmic form

$$J = \ln q(0) - \ln q(l), \tag{10}$$

where

$$\begin{aligned} & q(0) = q_n^{(1)}(0) + q_p^{(1)}(0) - \left(\frac{r_n}{l}\sum_{k=1}^{N_n} j_k + \frac{r_p}{l}\sum_{k=1}^{N_p} j_k\right), \\ & q(l) = q_n^{(N_n)}(1) + q_p^{(N_p)}(1) + \left(\frac{r_n}{l}\sum_{k=1}^{N_n} j_k + \frac{r_p}{l}\sum_{k=1}^{N_p} j_k\right). \end{aligned} \tag{11}$$

Functional (10) depends on parameters of specific current density j_k and impurity concentration C_k in sections of materials of which thermoelement is composed. Methods of optimal control theory allow purposeful search for optimal values of these parameters. The essence of this search reduces to the following. The Hamiltonian function is written as

$$H_{n,p}^k = (\psi_1^k f_1^k + \psi_2^k f_2^k)_{n,p}, \tag{12}$$

where $(f_1^k, f_2^k)_{n,p}$ – right-hand sides of Eq. (3). The vector-function of pulses $\psi = (\psi_1^k, \psi_2^k)_{n,p}$ should satisfy the system [16]

$$\left.\begin{aligned} \frac{d\psi_1^k}{dy_k} &= \frac{\alpha_k j_k}{\kappa_k} R_1^k \psi_1^k - \frac{\alpha_k^2 j_k}{\kappa_k} R_2^k \psi_2^k, \\ \frac{d\psi_2^k}{dy_k} &= \frac{j_k}{\kappa_k}\psi_1^k - \frac{\alpha_k j_k}{\kappa_k}\psi_2^k, \end{aligned}\right\}_{n,p}, \qquad (13)$$

where

$$\left.\begin{aligned} R_1^k &= 1 + \frac{d\ln\alpha_k}{dT}T - \frac{d\ln\kappa_k}{dT}\left(T + \frac{q}{\alpha_k}\right), \\ R_2^k &= R_1^k - \frac{1}{Z_k}\frac{d\ln\sigma_k}{dT} + \frac{d\ln\alpha_k}{dT}\left(T + \frac{q}{\alpha_k}\right), \end{aligned}\right\}_{n,p},$$

Z_k is figure of merit of material of kth section. The boundary conditions for system (13) are of the form

$$\psi_2^{(1)_{n,p}}(0) = \frac{1}{q(0)}, \quad \psi_2^{N_{n,p}}(1) = \frac{1}{q(l)},$$
$$\psi_1^{(k+1)_{n,p}}(0) = \psi_1^{(k)_{n,p}}(1), \quad \psi_2^{(k+1)_{n,p}}(0) = \psi_2^{(k)_{n,p}}(1), \quad k = 1, \ldots, N_{n,p} \qquad (14)$$

According to optimal control theory [17], to reach minimum functional J (10) (maximum efficiency) of segmented thermoelement, the following conditions should be met:

(1) Optimal values of specific current density for each thermoelement section $j_k^{n,p}$ should satisfy the equalities

$$-\left[\frac{\partial J}{\partial j_k}\right]_{n,p} + \int_0^1 \left[\psi_1^k \frac{\partial f_1^k}{\partial j_k} + \psi_2^k \frac{\partial f_2^k}{\partial j_k}\right]_{n,p} dy_k = 0, \quad k = 1, \ldots, N_{n,p}, \qquad (15)$$

that can be easily reduced to

$$j_k^{n,p} = \left(\frac{J_1^k}{2J_2^k - \partial J/\partial j_k}\right)_{n,p}, \quad k = 1, \ldots, N_{n,p}, \qquad (16)$$

where

$$\left.\begin{aligned} J_1^k &= \psi_2 q|_0^1 + \int_0^1 \frac{\alpha_k j_k}{\kappa_k}(\psi_1 T - \psi_2 q)dy, \\ J_2^k &= \int_0^1 \frac{\psi_2}{\sigma_k}(1 + Z_k T)dy, \end{aligned}\right\} k = 1, \ldots, N_{n,p},$$
$$\frac{\partial J}{\partial j_k} = -\frac{r_{n,p}}{l}\left(\frac{1}{q(0)} + \frac{1}{q(l)}\right). \qquad (17)$$

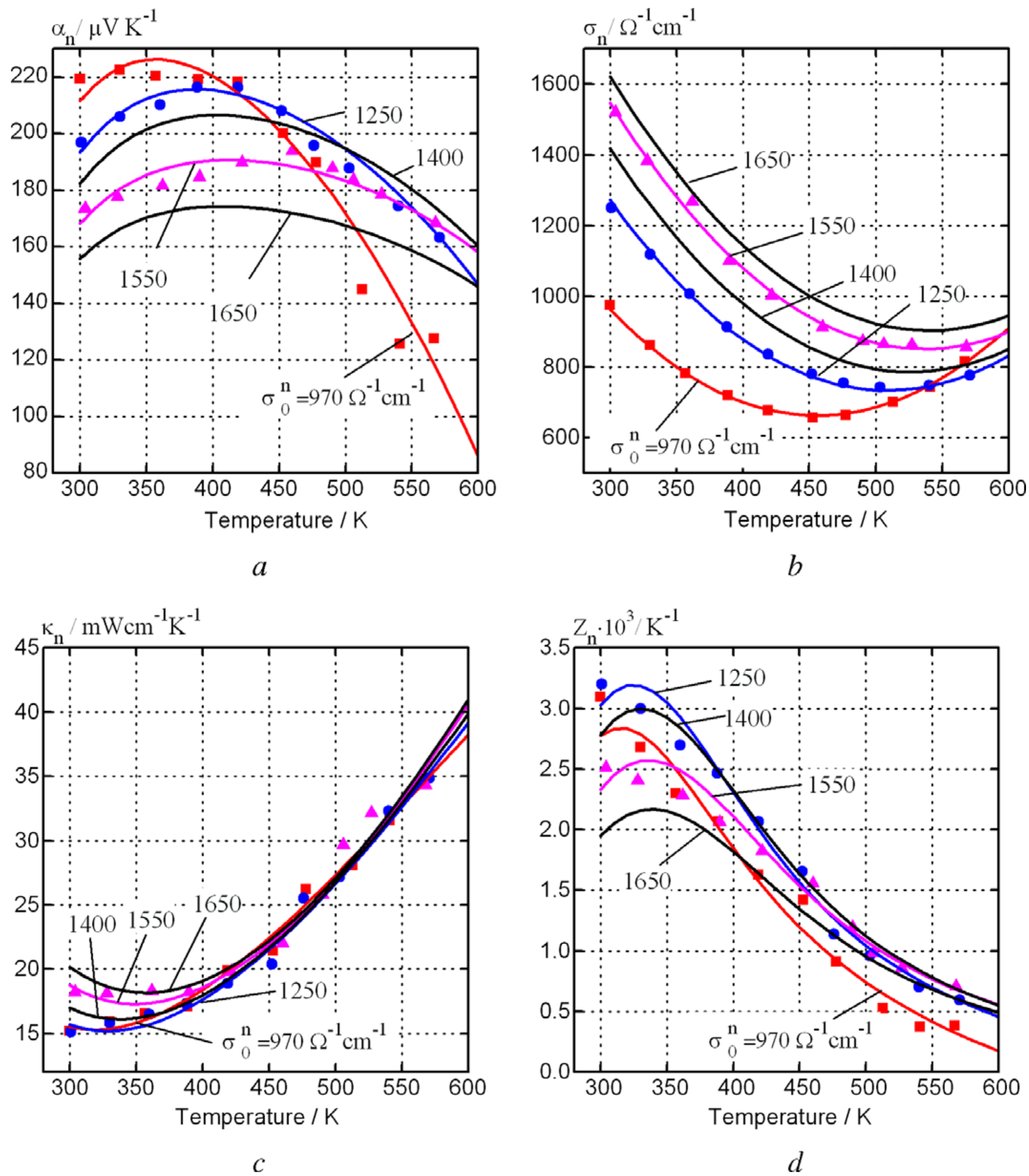

Fig. 2. Temperature dependences of α_n, σ_n, κ_n, and Z_n of alloy samples $(Bi_2Te_3)_{0.90}(Sb_2Te_3)_{0.05}(Sb_2Se_3)_{0.05}$ having different electric conductivity values σ_0^n under T = 300 K.

(2) Optimal values of impurity concentration in material of each section C_k are found from the relationships

$$\int_0^1 \left[\psi_1^k \frac{\partial f_1^k}{\partial C_k} + \psi_2^k \frac{\partial f_2^k}{\partial C_k}\right]_{n,p} dy = 0, \quad k = 1, \ldots, N_{n,p}. \tag{18}$$

To construct an algorithm of designing, the system of Eqs. (3)–(18) should be supplemented with functions relating material parameters α, σ, and κ to impurity concentration C and temperature T, moreover, one should sign permissible area of change in concentration G_C for materials of each section. It is reasonable to use experimental dependences $\alpha_k^{n,p} = \alpha_k^{n,p}(C_k^{n,p}, T)$, $\sigma_k^{n,p} = \sigma_k^{n,p}(C_k^{n,p}, T)$, and $\kappa_k^{n,p} = \kappa_k^{n,p}(C_k^{n,p}, T)$, approximating them by polynomials. Note, if a segmented thermoelement is designed of materials whose parameters are independent of impurity concentration, the optimality condition (18) is not used.

On determining optimal values of specific current density $j_k^{n,p}$, optimal parameters of thermoelement design, namely cross-sectional areas of its legs $S_{n,p}$ and heights of separate sections $l_k^{n,p}$, under given values of legs height l and strength of current I, passing through them ($I = \frac{W}{U}$, W, and U are electric power and voltage that should be provided by generator module with maximum efficiency), are calculated using the formulae.

$$S_{n,p} = \frac{Il}{\sum_{k=1}^{N_{n,p}} (j_k)_{n,p}}, \quad (l_k)_{n,p} = \frac{S_{n,p}}{I}(j_k)_{n,p}. \tag{19}$$

The number of thermoelements in a module to provide given value of voltage U is determined as

$$n_k = \frac{U}{q(l) - q(0)}, \tag{20}$$

where $q(l)$ and $q(0)$ are the values of specific (related to current strength) heat flows on thermoelement junctions corresponding to optimal values of control parameters $j_k^{n,p}$ and $C_k^{n,p}$.

In case when module construction is assigned, relationships (19) and (20) are used to determine optimal for maximum efficiency mode values of generated current I, voltage U and power W that can be anticipated on external loading.

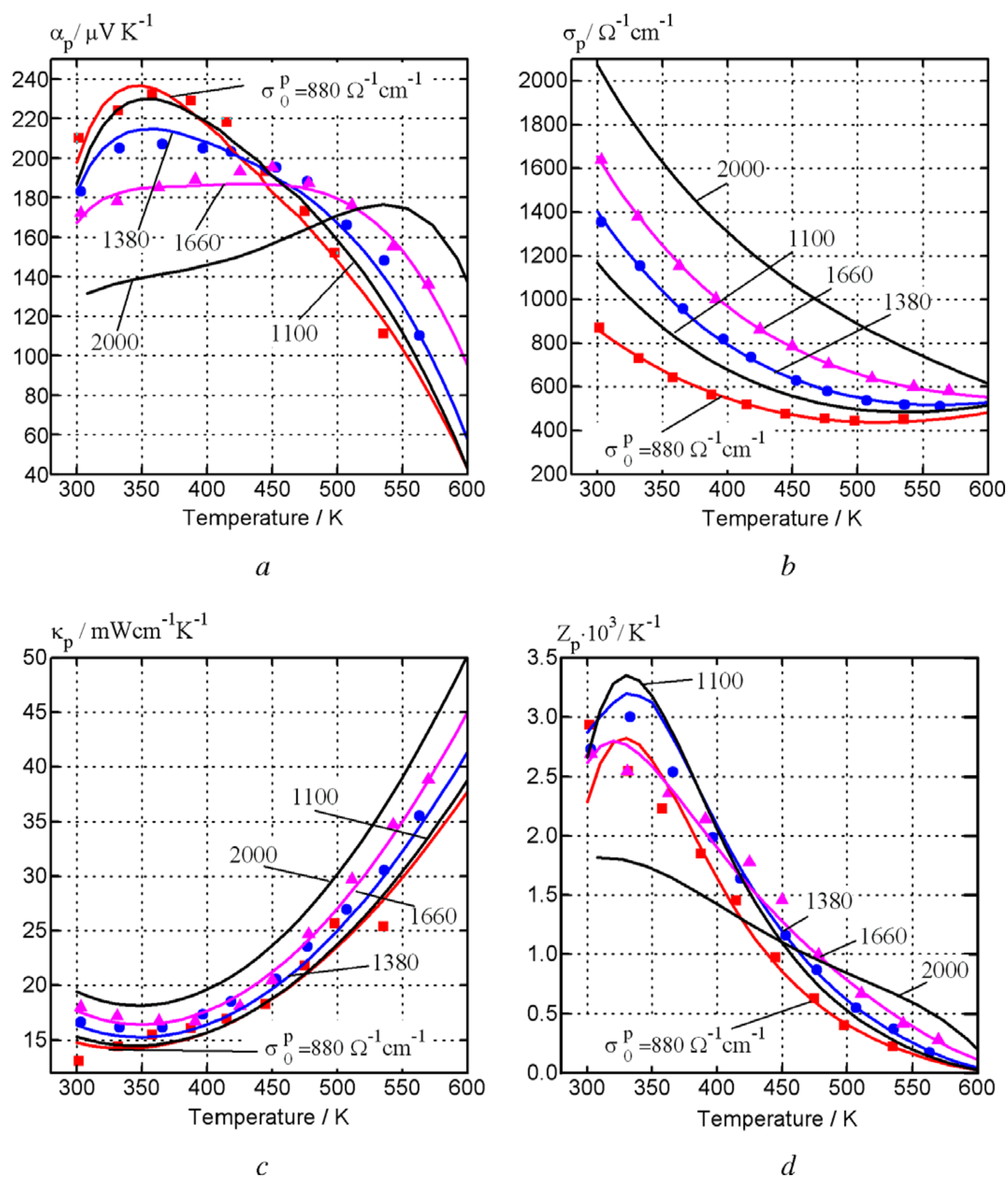

Fig. 3. Temperature dependences of α_p, σ_p, κ_p, and Z_p of alloy samples $(Bi_2Te_3)_{0.25}(Sb_2Te_3)_{0.72}(Sb_2Se_3)_{0.03}$ having different electric conductivity values σ_0^p under T = 300 K.

The system of Eqs. (3)–(20) forms the basis for the successive approximation algorithm of a computerized design of generator segmented thermoelements and modules with maximum efficiency based on them.

3. Results of computer design of generator modules and comparison with the experiment

The elaborated computerized method was used to design generator modules of BiTe-based materials. The main purpose of simulation is to estimate the possibility of increasing the efficiency of such modules by using for them concentration thermoelements of two-segmented legs.

The calculations used experimentally measured temperature dependences of parameters α, σ, and κ of zone meting-grown n-type $(Bi_2Te_3)_{0.90}(Sb_2Te_3)_{0.05}(Sb_2Se_3)_{0.05}$ and p-type $(Bi_2Te_3)_{0.25}$-$(Sb_2Te_3)_{0.72}(Sb_2Se_3)_{0.03}$ samples doped with iodine and lead, respectively. Figs. 2 and 3 show these dependences for samples with different degree of doping, hence, different concentration of current carriers, that can be characterized by electric conductivity value σ_0 under the temperature $T = 300$ K. These dependences were approximated by two-dimensional polynomials in the form of $\alpha^{n,p} = \alpha^{n,p}(\sigma_0^{n,p}, T), \sigma^{n,p} = \sigma^{n,p}(\sigma_0^{n,p}, T)$, and $\kappa^{n,p} = \kappa^{n,p}(\sigma_0^{n,p}, T)$, and polynomial coefficients were entered into computer program as the input data.

Figs. 2d and 3d demonstrate a change in figure of merit of materials with temperature. It can be seen that samples with high electric conductivity (up to $2000\,\Omega^{-1}\,\text{cm}^{-1}$), that is, more heavily doped, have increased figure of merit in the area of elevated temperatures. It is obvious that using for generators BiTe-based materials with electric conductivity exceeding $2000\,\Omega^{-1}\,\text{cm}^{-1}$ will not be efficient due to their low figure of merit.

Calculated optimal parameters values of materials under $T = 300$ K for single- and double-segmented legs of generator thermoelements with the operating temperature range 300–573 K are given in Table 1. The same table gives determined optimal temperatures at the boundary between the sections and segment heights for double-segmented thermoelements. The value of contact resistances in the calculations was assumed equal to $5 \times 10^{-6}\,\Omega\,\text{cm}^2$ on thermoelement junctions and $10^{-5}\,\Omega\,\text{cm}^2$ at the boundaries between the sections of legs.

In high-temperature segments, materials with increased electric conductivity and accordingly lower absolute value of the Seebeck coefficient should be used. When electric circuit is closed, current will be directed toward the Seebeck coefficient increase. Partial thermoEMFs caused by difference in α at the interface between the sections will be added, increasing thermoelement efficiency.

Calculated in maximum efficiency mode, power characteristics (optimal current, voltage, power, and efficiency) of modules of optimal material properties with the number of thermoelements $N_{TE} = 56$ couples, height of single-segmented legs 3 mm, double-segmented 5.6 mm and cross-sectional area 4.3×1.8 mm are summarized in Table 2. Using double-segmented legs for generator modules of BiTe-based materials, one can expect the efficiency at a level of 7.5%.

Table 2
Characteristics of generator modules of optimal BiTe-based materials under operating temperature range 300–573 K.

Module type	Generated electric power W, W	Current I, A	Voltage U, V	Efficiency η, %
Optimal characteristic values calculated in maximum efficiency mode				
Modules with single-segmented legs	14.6	4.6	3.19	6.6
Modules with double-segmented legs	8.9	2.6	3.44	7.5
Characteristic values obtained on experimental module samples				
Modules with single-segmented legs	10.1	4.4	2.30	6.5
Modules with double-segmented legs	6.5	2.55	2.55	7.3

Experimental module samples of the above-described design with single- and double-segmented legs were manufactured. For legs manufacturing, BiTe-based thermoelectric materials were selected with parameters α and σ_0 at $T = 300$ K close to calculated optimal values (Table 1). Results of measuring characteristics of these modules are given in Table 2. Experimentally obtained efficiency values of thermopiles are in good agreement with those theoretically predicted and testify to possible efficiency increase by nearly 15% by using BiTe-based modules with double-segmented legs instead of single-segmented.

At the same time, measured values of maximum voltage and, accordingly, power on loading, proved to be lower than theoretically calculated values. Power losses on experimental module samples are attributable to a deviation in individual thermoelectric legs of material parameter values of α, σ, and κ at $T = 300$ K and their temperature dependences from the calculated optimal values. When manufacturing generator modules, it is difficult to control conformity of material parameters α, σ, and κ to their optimal values for each leg. It is apparent that a more thorough choice of legs materials will allow reducing this discrepancy between the experimental and calculated characteristic values of generator modules.

4. Conclusions

Developed on the basis of optimal control theory, computerized method allows designing generator modules of segmented

Table 1
Parameters of BiTe-based materials ($T = 300$ K) for generator modules.

Type of leg	Calculated optimal parameter values				Range of parameter values in experimental module samples		Temperature at the segments interface, K	Segment height, mm
	σ_0, Ω^{-1} cm^{-1}	α, μV K^{-1}	κ, mW cm^{-1} K^{-1}	$Z\,10^3$, K^{-1}	σ_0, Ω^{-1} cm^{-1}	α, μV K^{-1}		
n-Type leg								
1 Segment	1370	186	16.5	2.9	1400–1450	190–180	–	3
2 Segments:								
Hot	1500	175	18.0	2.6	1500–1600	175–160	460	3.2
Cold	1300	194	16.0	3.0	1300–1400	200–190		2.4
p-Type leg								
1 Segment	1560	176	17.0	2.8	1600–1700	170–160	–	3
2 Segments:								
Hot	1980	123	19.0	1.3	1900–2000	130–110	457	3.2
Cold	1200	186	15.5	2.7	1200–1300	190–180		2.4

thermoelements, taking into account dependences of thermoelectric material parameters on temperature and current carrier concentration, electric, and thermal compatibility of segments, effect of contact resistances. This method has the advantage that it combines in one problem calculation of optimal material parameters for segments and determination of characteristics of modules of such materials.

The method was applied to design of generator modules of traditional thermoelectric materials based on BiTe. Theoretical estimation of maximum efficiency of modules with single- and double-segmented legs demonstrated the possibility of its increase by about 15% by using double-segmented thermoelements instead of single-segmented.

This conclusion was confirmed by the results of measuring characteristics of experimental generator samples. The efficiency of modules made of double-segmented legs reached 7.3%. Such efficiency level under operating temperatures from 300 K to 600 K provides further wide practical use of such modules in generators for heat recovery from organic fuel combustion [18], exhaust gases of engines [19–21], exhaust gases of garbage, coal and wood incineration furnaces [22,23], waste heat of electric power stations [24]. Such modules can be used as a low-temperature stage of thermoelectric generators of various terrestrial and space applications, utilizing heat of radioisotope sources [25] and solar radiation [26].

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